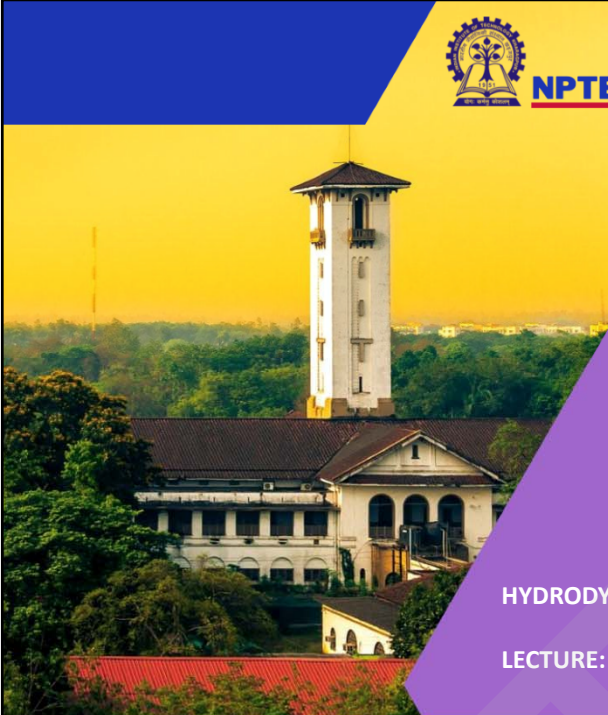




NPTEL ONLINE CERTIFICATION COURSE



SEMICONDUCTOR DEVICE MODELING AND SIMULATION

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HYDRODYNAMIC MODEL

LECTURE: 49



L49 HYDRONYMIC MODEL



- *As extension of DD model*

SEMICONDUCTOR DEVICE MODELING AND SIMULATION



LENGTH SCALES



l_ϕ = Phase coherence length = Distance an electron travels before suffering and phase change of 2π
 l_e = Elastic scattering length = Mean free path between elastic scattering events (Ion Impurity scattering)
 l_{in} = Inelastic scattering length = distance an electron travels before losing an energy $k_B T$ (Phonon scattering)

Diffusive Transport $L \gg l_e, \lambda_F$

Classical $\rightarrow l_\phi < l_e$
 Quantum $\rightarrow l_e < l_\phi \rightarrow T \downarrow$

Ballistic Transport $L < l_e, l_\phi$

Classical $L \gg \lambda_F$ (Fermi wavelength)
 Quantum $L \sim \lambda_F$

Fermi wavelength is proportional to the separation of the electrons

$$\lambda_F = \left(\frac{8\pi V}{3N} \right)^{1/3}$$

$\propto (1/n)^{1/3}$ metal $\rightarrow$ $n \sim 10^{22}$
 $\text{Si} \rightarrow 10^{10}$ $\text{Si} \rightarrow 10^{22}$
 $0.5 \text{ \AA} \rightarrow 7$
 $\lambda_F \uparrow$
 m^* $T \uparrow$

SEMICONDUCTOR DEVICE MODELING AND SIMULATION



RECALL DIFFUSIVE TRANSPORT



$$J_n = q(n\mu_n E + D_n \nabla n)$$

$\mu \propto D$

DD approach is valid and temperature dependent mobility is sufficient parameter

	Si	Ge	GaAs
Electrons $\propto$	$T^{-2.4}$	$T^{-1.7}$	$T^{-1.0}$
Holes $\propto$	$T^{-2.2}$	$T^{-2.3}$	$T^{-2.1}$

SEMICONDUCTOR DEVICE MODELING AND SIMULATION



MOSFET MOBILITY



Let I_D is the drain current, L is the length and W is the channel width, V_{GS} is the gate voltage, and V_T is the threshold gate voltage. **Effective mobility μ_{eff}** is deduced from the first-order one-dimensional model in the linear mode ($V_{DS} \ll V_{GS} - V_T$) using drain conductance (g_D)

$$\mu_{eff} \approx \frac{L g_D}{W C_{OX} (V_{GS} - V_T)}$$

$$g_D = \left. \frac{\partial I_D}{\partial V_{DS}} \right|_{V_{GS} = \text{const}}$$

Field effect mobility μ_{FE} is given by (g_m is trans-conductance)

$$\mu_{FE} = \frac{L g_m}{W C_{OX} V_{DS}}$$

$$g_m = \left. \frac{\partial I_D}{\partial V_{GS}} \right|_{V_{DS} = \text{const}}$$

experimentally measured $\mu_{FE} \ll \mu_{eff}$ due to neglect of the gate voltage dependence ($E_{\perp}$)

$$I_D = \mu_n C_{OX} \frac{W}{L} (V_{GS} - V_T) V_{DS} - \frac{\mu_n C_{OX} W}{2L} V_{DS}^2$$

Lin

$$= \mu_n C_{OX} \frac{W}{L} (V_{GS} - V_T)^2$$

sat

$$g_m = \frac{\partial I_D}{\partial V_{GS}} = \mu_n C_{OX} \frac{W}{L} (V_{GS} - V_T)$$

SEMICONDUCTOR DEVICE MODELING AND SIMULATION



MOSFET MOBILITY



μ_{FE} is related to μ_{eff}

$$\mu_{FE} \approx \mu_{eff} + (V_{GS} - V_T) \frac{\partial \mu_{eff}}{\partial V_{GS}} \bigg|_{V_{DS} = \text{const}}$$

$$\frac{\partial \mu_{eff}}{\partial V_{GS}} = \frac{L}{W C_{OX}} \left(\frac{\frac{\partial g_D}{\partial V_{GS}}}{V_{GS} - V_T} - \frac{g_D}{(V_{GS} - V_T)^2} \right)$$

effective mobility decreases with the gate voltage $\frac{\partial \mu_{eff}}{\partial V_{GS}} < 0$, except for very low voltage, where mobility increase due to decreased coulombic scattering

Saturation mobility μ_{sat} is deduced from saturation characteristics

$$I_{D,sat} = \frac{W \mu_{sat} C_{OX}}{2L} (V_{GS} - V_T)^2$$

$$A = \frac{L}{W C_{OX}} \frac{\partial}{\partial V_{GS}} \left(\frac{\partial I_{D,sat}}{\partial V_{GS}} \right)$$

$$= \frac{L}{W C_{OX}} \frac{\partial}{\partial V_{GS}} \left(\frac{\partial I_{D,sat}}{\partial V_{GS}} \right)$$

experimentally μ_{sat} are always smaller compared to μ_{eff}

$$\mu_{sat} \approx \mu_{eff} \left(\frac{V_{GS} - V_T}{V_{GS}} \right)$$

SEMICONDUCTOR DEVICE MODELING AND SIMULATION



FIELD DEPENDENT MOBILITY



$$J_n = q(n\mu_n E + D_n \nabla n)$$

For short channel devices, high field exists

High field

- $v_s \sim$ saturation velocity
- E_p = optical-phonon energy
- Constant $\alpha \sim 2$ (electron)
- ~ 1 (holes)

$$v_d = \frac{\mu_0 E}{\left[1 + \left(\frac{\mu_0 E}{v_{sat}}\right)^\alpha\right]^{1/\alpha}}, \quad v_s = \sqrt{\frac{8E_p}{3\pi m_0}} \approx 10^7 \text{ cm/s}$$

SEMICONDUCTOR DEVICE MODELING AND SIMULATION

Handwritten notes and derivations:

$$\mu(E) = \frac{\mu_0}{\left(1 + \left(\frac{\mu_0 E}{v_s}\right)^\alpha\right)^{1/\alpha}}$$

$$\frac{\partial \langle E \rangle}{\partial t} = e E v_n - \frac{E_{op}}{\tau_e} \rightarrow 0$$

$$\frac{\partial \langle m^* v_s \rangle}{\partial t} = e E - \frac{m^* v_s}{\tau_m} \rightarrow 0$$

$$v_s^2 = \frac{E_{op}}{m^*} \rightarrow v_s = \sqrt{\frac{E_{op}}{m^*}}$$

For tetrahedral semiconductors:
 $E_{op} \sim 40 \text{ meV}$
 $m^* \sim 0.1 m_0$



SATURATION VELOCITY



Energy loss rate

$$\frac{d \langle E \rangle}{dt} = e \xi v_s - \frac{E_{op}}{\tau_e}$$

Momentum relaxation

$$\frac{d \langle m^* v_s \rangle}{dt} = e \xi - \frac{m^* v_s}{\tau_m}$$

For tetrahedral semiconductors
 E_{op} = optical-phonon energy $\sim 40 \text{ meV}$
 $m^* \sim 0.1 m_0$

Handwritten note:

$$v_s \propto \sqrt{\frac{E_{op}}{m^*}}$$

SEMICONDUCTOR DEVICE MODELING AND SIMULATION



VELOCITY OVERSHOOT

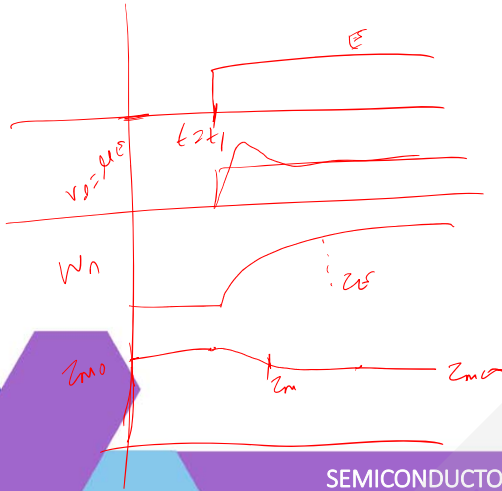
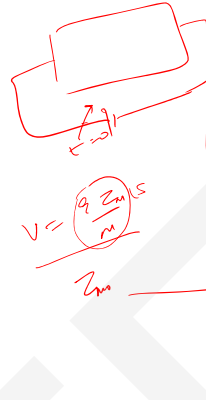
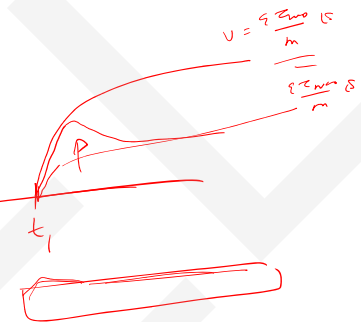


Temporal - sudden step in the electric field as a function of time

- Momentum relaxation time decreases as function of time
- Carrier energy increases as function of time

$$\tau_s > \tau_m$$

$$v_d = \frac{q \langle \tau \rangle}{m} E$$



SEMICONDUCTOR DEVICE MODELING AND SIMULATION



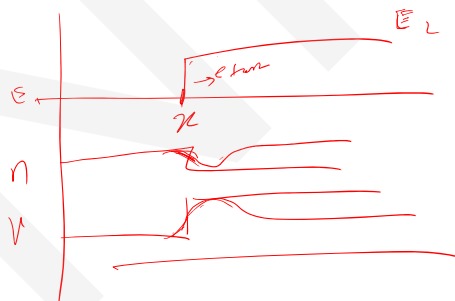
VELOCITY OVERSHOOT



Spatial - sudden step in electric field at particular location (steady state, $J = \text{const}$)

- Momentum relaxation time decreases as function of position
- Carrier energy increases as function of position

$$v_d = \frac{q \langle \tau \rangle}{m} E$$



$$J = q n v$$

$$\tau_m \rightarrow \text{decreases}$$

$$\tau_s \rightarrow \text{increases}$$

SEMICONDUCTOR DEVICE MODELING AND SIMULATION

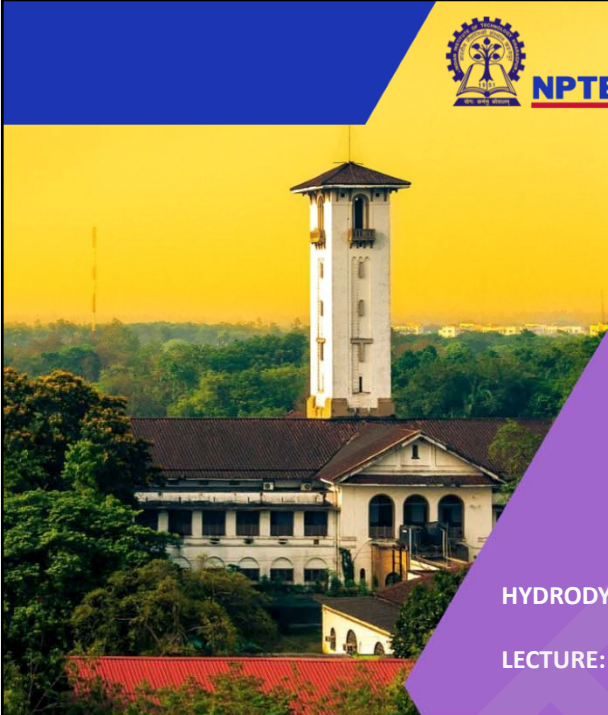


Summary



- *Velocity and Non-local effects discussed*
- *Discussed ways to extend the validity of DD model*

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HYDRODYNAMIC MODEL

LECTURE: 50



L50 HYDRONYMIC MODEL



- *Carrier Balance, Energy balance and momentum balance Equations*

SEMICONDUCTOR DEVICE MODELING AND SIMULATION



HYDRODYNAMIC MODEL THEORY



- Basic hydrodynamic (balance) equations - derivation $\frac{\partial f}{\partial t} + \frac{\partial f}{\partial r} \frac{\partial r}{\partial k} + \frac{\partial f}{\partial k} \frac{\partial k}{\partial r} = \left(\frac{\partial f}{\partial t}\right)_{coll}$
- Ensemble relaxation rates and their calculation $\frac{\partial f}{\partial t} = -\frac{1}{\hbar} \nabla_k E \cdot \nabla_r f - \frac{\mathbf{F}_e}{\hbar} \cdot \nabla_k f + \left(\frac{\partial f}{\partial t}\right)_{coll}$ $\frac{\partial f}{\partial t} = -\frac{f - f_0}{\tau}$
- Recall BTE $\frac{\partial f}{\partial t} = -\frac{1}{\hbar} \nabla_k E \cdot \nabla_r f - \frac{\mathbf{F}_e}{\hbar} \cdot \nabla_k f + \left(\frac{\partial f}{\partial t}\right)_{coll}$ $\frac{\partial f}{\partial t} = -\frac{f - f_0}{\tau}$
- Multiply BTE with an appropriate function $\phi(k)$ and integrate over k

$$\frac{\partial \int \phi \cdot f \cdot d\vec{k}}{\partial t} = -\nabla_r \cdot \left(\int \phi \cdot \vec{u} \cdot f \cdot d\vec{k} \right) + \frac{\mathbf{F}_e}{\hbar} \cdot \int (\nabla_k \phi) \cdot f \cdot d\vec{k} + \int \left(\frac{\partial f}{\partial t} \right)_{coll} \phi \cdot d\vec{k}$$

$$\int \phi \frac{\partial f}{\partial t} = -\frac{1}{\hbar} \int \phi \frac{\partial E}{\partial k} \frac{\partial f}{\partial r} - \frac{\mathbf{F}_e}{\hbar} \int \phi \frac{\partial f}{\partial k} + \int \left(\frac{\partial f}{\partial t} \right)_{coll} \phi$$

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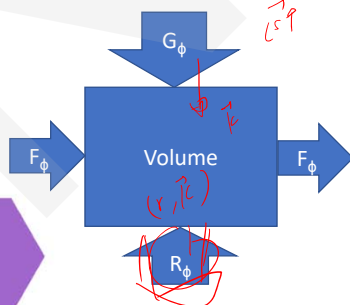


BALANCE EQUATIONS



$$\frac{\partial \int \phi \cdot f \cdot d\vec{k}}{\partial t} = -\nabla_r \cdot \left(\int \phi \cdot \vec{u} \cdot f \cdot d\vec{k} \right) + \frac{\mathbf{F}_e}{\hbar} \cdot \int (\nabla_k \phi) \cdot f \cdot d\vec{k} + \int \left(\frac{\partial f}{\partial t} \right)_{coll} \phi \cdot d\vec{k}$$

$$\frac{\partial n_\phi}{\partial t} = -\nabla_r F_\phi + G_\phi - R_\phi$$



$$n_\phi = \int \phi \cdot f \cdot d\vec{k}$$

$$F_\phi = \int \phi \cdot \vec{u} \cdot f \cdot d\vec{k}$$

$$G_\phi = \frac{\mathbf{F}_e}{\hbar} \cdot \int (\nabla_k \phi) \cdot f \cdot d\vec{k}$$

$$R_\phi = -\int \left(\frac{\partial f}{\partial t} \right)_{coll} \phi \cdot d\vec{k}$$

SEMICONDUCTOR DEVICE MODELING AND SIMULATION



MOMENTUM BALANCE EQUATIONS



Balance equation for the momentum: $\phi(k) = -q\hbar k/m^* = -q\hbar k/m^*$: $\phi(k) = \vec{u} = \frac{\vec{p}}{m}$

$$n_\phi = \int \phi \cdot f \cdot d\vec{k} = J_n$$

$$F_\phi = \int \phi \cdot \vec{u} \cdot f \cdot d\vec{k} = -2qW_n / m^*$$

$$G_\phi = \frac{\vec{F}_e}{\hbar} \cdot \int (\nabla_k \phi) \cdot f \cdot d\vec{k} = q^2 n \vec{E} / m^*$$

$$R_\phi = -\int \left(\frac{\partial f}{\partial t} \right)_{\text{coll}} \phi \cdot d\vec{k} = \frac{J_n}{\tau_M}$$

$$\frac{\partial (J_n)}{\partial t} = \frac{2q}{m^*} \nabla \cdot W_n + q^2 n \vec{E} / m^* - \frac{J_n}{\tau_M}$$

$$\begin{aligned} \frac{\partial (n \vec{v}_n)}{\partial t} + \frac{qn}{m_n^*} \vec{E} + \frac{1}{m_n^*} \cdot \frac{\partial (n k_B T_n)}{\partial r} &= -\frac{n \vec{v}_n}{\tau_n} = -\frac{J_n}{\tau_n} \\ \frac{\partial (p \vec{v}_p)}{\partial t} - \frac{qp}{m_p^*} \vec{E} + \frac{1}{m_p^*} \cdot \frac{\partial (p k_B T_p)}{\partial r} &= -\frac{p \vec{v}_p}{\tau_p} \end{aligned}$$

Handwritten notes: $\frac{\partial n}{\partial r} + \frac{\partial T}{\partial r}$, $n k_B T_e$, $\frac{\partial n}{\partial r} + \frac{\partial T}{\partial r}$

SEMICONDUCTOR DEVICE MODELING AND SIMULATION



ENERGY BALANCE EQUATIONS



Balance equation for the energy: $\phi(k) = \hbar^2 k^2 / 2m^*$: $\frac{1}{2} m u^2$

$$n_\phi = \int \phi \cdot f \cdot d\vec{k} = W_n$$

$$F_\phi = \int \phi \cdot \vec{u} \cdot f \cdot d\vec{k} = W_w$$

$$G_\phi = \frac{\vec{F}_e}{\hbar} \cdot \int (\nabla_k \phi) \cdot f \cdot d\vec{k} = \vec{E} \cdot J_n$$

$$R_\phi = -\int \left(\frac{\partial f}{\partial t} \right)_{\text{coll}} \phi \cdot d\vec{k} = \frac{W_n - W_0}{\tau_E}$$

Final form of the energy balance equation:

$$\frac{\partial (W_n)}{\partial t} = -\nabla \cdot (F_w) + \vec{E} \cdot J_n - \frac{W_n - W_0}{\tau_E} + E_g (R - G)$$

Handwritten notes: $\frac{W_n - W_0}{\tau_E}$, $\frac{W_n - W_0}{\tau_E}$, $\frac{W_n - W_0}{\tau_E}$, $\frac{W_n - W_0}{\tau_E}$, $\frac{W_n - W_0}{\tau_E}$, $\frac{W_n - W_0}{\tau_E}$, $\frac{W_n - W_0}{\tau_E}$, $\frac{W_n - W_0}{\tau_E}$, $\frac{W_n - W_0}{\tau_E}$, $\frac{W_n - W_0}{\tau_E}$

SEMICONDUCTOR DEVICE MODELING AND SIMULATION



CLOSURE



- Each new balance equation introduces new variable
- Carrier concentration $\rightarrow$ carrier flux or momentum $\rightarrow$ momentum flux or Kinetic energy $\rightarrow$ energy flux
- To have a closed set of equations, one either:
 - (a) ignores energy (heat) flux altogether
 - (b) use a phenomenological "closure" relation for heat flux

SEMICONDUCTOR DEVICE MODELING AND SIMULATION



COMPLETE BALANCE EQUATIONS



CB $\frac{\partial n}{\partial t} = \frac{1}{q} \nabla \cdot (J_n) + \left(\frac{\partial n}{\partial t} \right)_{coll}$

MB $\frac{\partial (J_n)}{\partial t} = \frac{2q}{m^*} \nabla \cdot W_n + q^2 n \bar{E} / m^* - \frac{J_n}{\tau_M}$

EB $\frac{\partial (W_n)}{\partial t} = -\nabla \cdot (F_W) + \bar{E} \cdot \bar{J}_n - \frac{W_n - W_0}{\tau_E}$

HEAT $\rho c \frac{\partial T_L}{\partial t} = \nabla \cdot [k(T) \nabla T_L] + E \cdot J + E_g [R - G]$

Handwritten notes:
 - A bracket groups the CB, MB, and EB equations, with a note "Poisson $\rightarrow$ $\frac{E}{q}$ ".
 - Under the HEAT equation, "Thermal cond" is written with an arrow pointing to the $\frac{\partial T_L}{\partial t}$ term.
 - The τ_M and τ_E terms in the MB and EB equations are underlined.

SEMICONDUCTOR DEVICE MODELING AND SIMULATION



DD AS A SPECIAL CASE



CB $\frac{\partial n}{\partial t} = \frac{1}{q} \nabla \cdot (J_n) + \left(\frac{\partial n}{\partial t} \right)_{\text{coll}}$ → $\frac{\partial n}{\partial t}$ → 2 Cont. → 2 } DD

MB $\frac{\partial (J_n)}{\partial t} = \frac{2q}{m^*} \nabla \cdot W_n + q^2 n \bar{E} / m^* - \frac{J_n}{\tau_M}$ → $J_n = \tau_M \left(\frac{\partial n}{\partial t} + \frac{J_n}{\tau_M} \right)$ → 2

EB $\frac{\partial (W_n)}{\partial t} = -\nabla \cdot (F_w) + \bar{E} \cdot \bar{J}_n - \frac{W_n - W_0}{\tau_E}$

HEAT $\rho c \frac{\partial T_L}{\partial t} = \nabla \cdot [k(T) \nabla T_L] + E \cdot J + E_g [R - G]$

SEMICONDUCTOR DEVICE MODELING AND SIMULATION

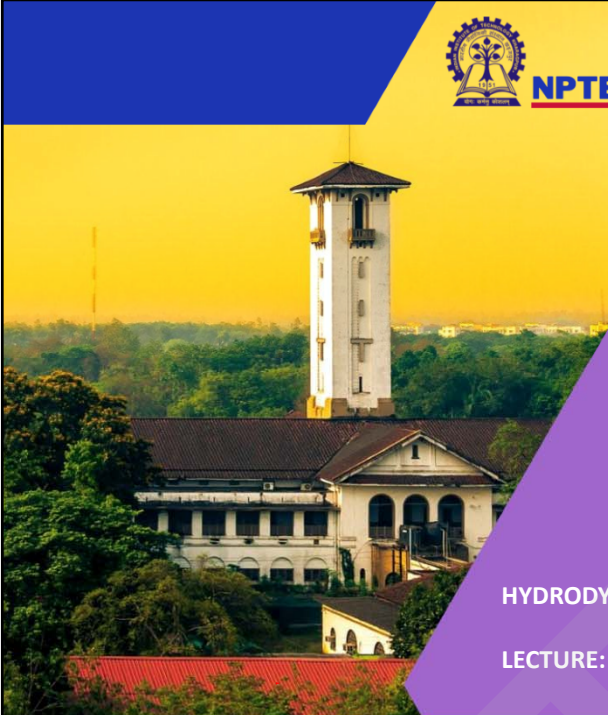


CONCLUSION



- Derived and discussed balance equations for hydrodynamic model.

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SEMICONDUCTOR DEVICE MODELING AND SIMULATION

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HYDRODYNAMIC MODEL

LECTURE: 51



L5I HYDRONYMIC MODEL



- *Relaxation times and Numerical schemes*

SEMICONDUCTOR DEVICE MODELING AND SIMULATION



SEMI-CLASSICAL APPROACH



1. Drift-Diffusion Method OR local field model: $\mu = f(\text{local field}) \Rightarrow v = \mu E$
2. Hydrodynamic/energy balance OR local energy model: $\mu = f(\text{local energy})$
3. Direct Solution of the Boltzmann Transport Equation via:
 - Particle-Based Approaches – Monte Carlo method
 - Numerical Solution to Boltzmann-Poisson Problem
4. Quantum transport:
 - Schrodinger-Poisson
 - density gradient / quantum potential.

SEMICONDUCTOR DEVICE MODELING AND SIMULATION



ENSEMBLE RELAXATION RATE



Ensemble relaxation rate (appeared in carrier balance equation) is related to intervalley transfer

$$\left(\frac{\partial n}{\partial t} \right)_{\text{coll}} = - \frac{n - n_0}{\tau_n}$$

CB $\rightarrow$ τ_1 $\uparrow$ n_g
 MB $\rightarrow$ τ_m $\downarrow$
 EB $\rightarrow$ τ_s $\downarrow$

These rates are calculated using Monte Carlo simulations.

Recall,

$$R_\phi = - \int \left(\frac{\partial f}{\partial t} \right)_{\text{coll}} \phi \cdot d\vec{k} = \frac{n_\phi - n_\phi^0}{\tau_\phi}$$

relaxation rate

$$\frac{1}{\tau_\phi} = \int_{V_k} \left[1 - \frac{f(k')S(k',k)}{f(k)S(k,k')} \right] S(k,k') dk'$$

$$\left(\frac{\partial f}{\partial t} \right)_{\text{coll}} = \int_{k'} [f(k')S(k',k) - f(k)S(k,k')] dk'$$

$$R_\phi = \int_{V_k} \int_{V_{k'}} [f(k)S(k,k') - f(k')S(k',k)] \phi(k) dk'$$

$$R_\phi = \int_{V_k} \left[\int_{V_{k'}} \left[1 - \frac{f(k')S(k',k)}{f(k)S(k,k')} \right] S(k,k') dk' \right] \phi(k) f(k) dk$$

SEMICONDUCTOR DEVICE MODELING AND SIMULATION



ENSEMBLE RELAXATION RATE



Ensemble relaxation rate (appeared in carrier balance equation) is related to intervalley transfer

$$\left(\frac{\partial n}{\partial t} \right)_{coll} = - \frac{n - n_0}{\tau_n}$$

These rates are calculated using Monte Carlo simulations.

Recall,

$$R_\phi = - \int \left(\frac{\partial f}{\partial t} \right)_{coll} \phi \cdot d\vec{k} = \frac{n_\phi - n_\phi^0}{\tau_\phi}$$

Ensemble relaxation rate

$$\left\langle \frac{1}{\tau_\phi} \right\rangle = \frac{R_\phi}{n_\phi - n_\phi^0}$$

all three

SEMICONDUCTOR DEVICE MODELING AND SIMULATION



MOMENTUM RELAXATION RATE



Recall momentum balance equation

$$\frac{\partial (J_n)}{\partial t} = \frac{2q}{m^*} \nabla \cdot W_n + q^2 n \vec{E} / m^* - \frac{J_n}{\tau_M} + 0$$

Under steady state, for a bulk semiconductor with uniform electric field

$$\frac{\partial (v_d)}{\partial t} = q \vec{E} / m^* - \frac{v_d}{\tau_M} = 0$$

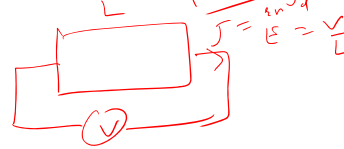
momentum relaxation rate

$$\frac{1}{\tau_M} = \frac{q \vec{E}}{m^* v_d}$$

$$J_n v_d = J_n = \frac{q^2 n E \tau_M}{m^*} = n q \tau_M (q E)$$

$$\mu = \frac{q \tau_M}{m^*} = n q \mu$$

$$\mu = \frac{q \tau_M}{m^*}$$



SEMICONDUCTOR DEVICE MODELING AND SIMULATION



DISCRETIZATION OF HD EQUATION



balance equations have following form

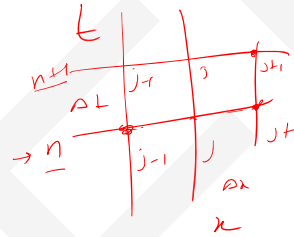
$$\frac{\partial u}{\partial t} = - \frac{\partial F(u)}{\partial x} = -v \frac{\partial u}{\partial x}$$

$$\frac{\partial u}{\partial t} = \frac{u(t+\Delta t) - u(t)}{\Delta t}$$

$$\frac{\partial u}{\partial x} = \frac{u(x+\Delta x) - u(x-\Delta x)}{2\Delta x}$$

Discretized Forward Time Centered Space (FTCS) form

$$\frac{u_j^{n+1} - u_j^n}{\Delta t} = -v \left(\frac{u_{j+1}^n - u_{j-1}^n}{2\Delta x} \right)$$



SEMICONDUCTOR DEVICE MODELING AND SIMULATION



VON NEUMANN STABILITY



It is local. Assuming coefficients of the difference equations sufficiently slowly varying as to be considered constant in space and time.

→ independent solutions, or eigenmodes, of the difference equations are all of the form

$$u_j^n = \xi^n e^{ikj\Delta x}$$

Handwritten notes: $u_j^n = u(x=j\Delta x, t=n\Delta t)$, $\xi \rightarrow \xi^n \rightarrow \xi^{n\Delta t} \rightarrow e^{i(k\Delta x)n\Delta t} \rightarrow e^{i2\pi k n \Delta x \Delta t}$

→ difference equation is unstable (exponentially growing modes) if amplification factor, $|\xi(k)| > 1$ for some k .

For FTCS

$$\frac{u_j^{n+1} - u_j^n}{\Delta t} = -v \left(\frac{u_{j+1}^n - u_{j-1}^n}{2\Delta x} \right)$$

$$\xi^{n+1} e^{i(k\Delta x)(j+1)} - \xi^n e^{i(k\Delta x)j} = -v \left(\frac{\xi^n e^{i(k\Delta x)(j+1)} - \xi^n e^{i(k\Delta x)(j-1)}}{2\Delta x} \right) \Delta t$$

$$\xi - 1 = -v \left(\frac{e^{i(k\Delta x)} - e^{-i(k\Delta x)}}{2\Delta x} \right) \Delta t$$

$$\xi = 1 - i \frac{v\Delta t}{\Delta x} \sin(k\Delta x)$$

→ unstable $\rightarrow \xi = 1 - \frac{v\Delta t}{\Delta x} \sin(k\Delta x)$

$$\xi(k) = 1 - i \frac{v\Delta t}{\Delta x} \sin(k\Delta x)$$

SEMICONDUCTOR DEVICE MODELING AND SIMULATION



LAX METHOD



Lax Method

$$\frac{u_j^{n+1} - u_j^n}{\Delta t} = -v \left(\frac{u_{j+1}^n - u_{j-1}^n}{2\Delta x} \right)$$

$$u_j^n = \frac{u_{j+1}^n + u_{j-1}^n}{2}$$

$$u_j^{n+1} = \frac{1}{2}(u_{j+1}^n + u_{j-1}^n) - \frac{v\Delta t}{2\Delta x}(u_{j+1}^n - u_{j-1}^n)$$

$$e^{ik\Delta x} + e^{-ik\Delta x}$$

Stability Criterion

$$\xi(k) = \cos(k\Delta x) - i \frac{v\Delta t}{\Delta x} \sin(k\Delta x)$$

Courant condition

$$\frac{|v| \Delta t}{\Delta x} \leq 1$$

$$|\xi(k)| = \cos^2(k\Delta x) + \left(\frac{v\Delta t}{\Delta x}\right)^2 \sin^2(k\Delta x) \leq 1$$

$$|v| \leq \frac{\Delta x}{\Delta t}$$

SEMICONDUCTOR DEVICE MODELING AND SIMULATION



PHASE ERROR



Lax Method
$$u_j^{n+1} = \frac{1}{2}(u_{j+1}^n + u_{j-1}^n) - \frac{v\Delta t}{2\Delta x}(u_{j+1}^n - u_{j-1}^n)$$

Stability Criterion

$$\xi(k) = \cos(k\Delta x) - i \frac{v\Delta t}{\Delta x} \sin(k\Delta x)$$

$$= e^{-ik\Delta x} + i \left(1 - \frac{v\Delta t}{\Delta x}\right) \sin(k\Delta x)$$

Phase error=0 if
$$\frac{|v| \Delta t}{\Delta x} = 1$$

$$\Delta t \sim \frac{\Delta x}{v}$$

SEMICONDUCTOR DEVICE MODELING AND SIMULATION



STABLE METHODS



First order Euler: explicit, stable for $r = \Delta t / k (\Delta x)^2 \leq 0.5$

$$\frac{\Phi_j^{n+1} - \Phi_j^n}{\Delta t} = \frac{\Phi_{j+1}^n - 2\Phi_j^n + \Phi_{j-1}^n}{k(\Delta x)^2}$$

Handwritten notes: $\xi^{n+1} e^{i k j \Delta x} - \xi^n e^{i k j \Delta x} = \frac{\xi^n e^{i k (j+1) \Delta x} - 2 \xi^n e^{i k j \Delta x} + \xi^n e^{i k (j-1) \Delta x}}{k \Delta x^2}$
 $\xi^n e^{i k j \Delta x} \Rightarrow \frac{\xi^n - 1}{\Delta t} = \frac{(e^{i k \Delta x} - 2 + e^{-i k \Delta x})}{k \Delta x^2}$

Crank Nicholson: implicit, always stable

$$\frac{\Phi_j^{n+1} - \Phi_j^n}{\Delta t} = \frac{\Phi_{j+1}^{n+1} - 2\Phi_j^{n+1} + \Phi_{j-1}^{n+1}}{2k(\Delta x)^2} + \frac{\Phi_{j+1}^n - 2\Phi_j^n + \Phi_{j-1}^n}{2k(\Delta x)^2}$$

Handwritten notes: $\xi = 1 + \frac{\Delta t}{k \Delta x^2} (2 \cos k \Delta x - 2)$
 $|\xi| = \left| 1 + \frac{\Delta t}{k \Delta x^2} (2 \cos k \Delta x - 2) \right| \leq 1$
 $\Rightarrow \frac{\Delta t}{k \Delta x^2} \times 2 \times 2 \sin^2 \frac{k \Delta x}{2} \leq 1$
 $\frac{\Delta t}{k \Delta x^2} \leq \frac{1}{2}$
 $\xi = \frac{1 - \frac{\Delta t}{2k \Delta x^2} 2 \sin^2 \frac{k \Delta x}{2}}{1 - \frac{\Delta t}{2k \Delta x^2} 4 \sin^2 \frac{k \Delta x}{2}} = 1$

SEMICONDUCTOR DEVICE MODELING AND SIMULATION



STABLE METHODS



DuFort-Frankel scheme: explicit, unconditionally stable

$$\frac{\Phi_j^{n+1} - \Phi_j^{n-1}}{2\Delta t} = \frac{\Phi_{j+1}^n - \Phi_j^{n+1} - \Phi_j^{n-1} + \Phi_{j-1}^n}{k(\Delta x)^2}$$

$$\frac{\partial \psi_p}{\partial t} = \nabla_r F + G_p - R_p + S$$

Leapfrog scheme: explicit, always unstable

$$\frac{\Phi_j^{n+1} - \Phi_j^{n-1}}{2\Delta t} = \frac{\Phi_{j+1}^n - 2\Phi_j^n + \Phi_{j-1}^n}{k(\Delta x)^2}$$

$$\xi^n \rightarrow 0 \text{ as } n \rightarrow \infty$$

SEMICONDUCTOR DEVICE MODELING AND SIMULATION



CONCLUSION



- Discussed the relaxation time and discretization of Hydrodynamic equations.

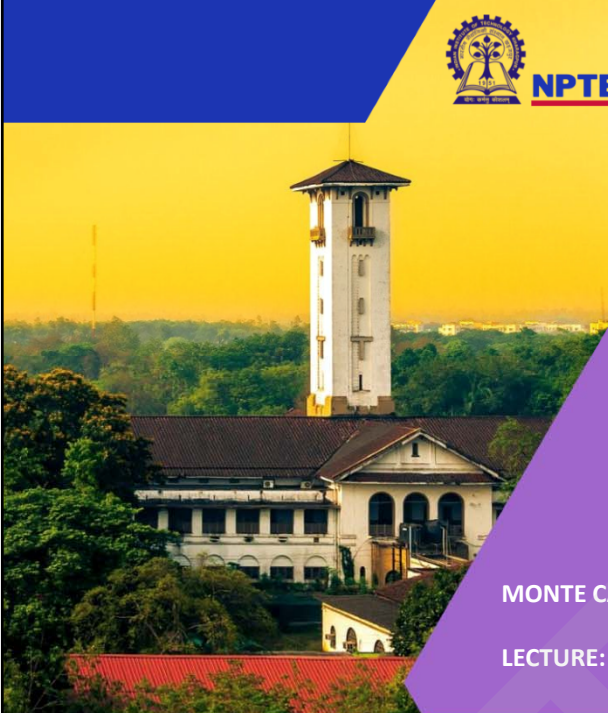
SEMICONDUCTOR DEVICE MODELING AND SIMULATION



Thank you



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SEMICONDUCTOR DEVICE MODELING AND SIMULATION

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MONTE CARLO SIMULATIONS

LECTURE: 52



L52 MONTE CARLO SIMULATIONS



- Monte Carlo simulations

BTE
H) → p-emission →

SEMICONDUCTOR DEVICE MODELING AND SIMULATION



RECALL BTE



- Recall BTE,
$$\frac{\partial f}{\partial t} + \frac{1}{\hbar} \nabla_k E \cdot \nabla_r f + \frac{\mathbf{F}_e}{\hbar} \cdot \nabla_k f = \left(\frac{\partial f}{\partial t} \right)_{coll}$$

$$\frac{\partial f}{\partial t} = \frac{V}{8\pi^3} \int d\mathbf{k} \{ f_{k'} [1 - f_k] S(k', k) - f_k [1 - f_{k'}] S(k, k') \} = - \frac{f - f_0}{\tau}$$

- BTE is a complicated integro-differential equations for distribution functions f , which is usually solved by
 - Making simplifying approximations; Drift-Diffusion (DD) and Hydrodynamic (HD) approaches
 - Statistical approach: Monte carlo simulations (MC)

SEMICONDUCTOR DEVICE MODELING AND SIMULATION



BTE



- Recall BTE,
$$\frac{\partial f}{\partial t} + \frac{1}{\hbar} \nabla_k E \cdot \nabla_r f + \frac{\mathbf{F}_e}{\hbar} \cdot \nabla_k f = \left(\frac{\partial f}{\partial t} \right)_{coll}$$

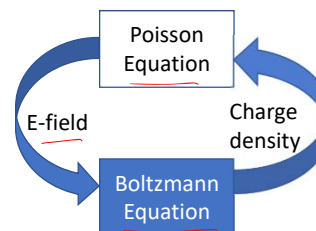
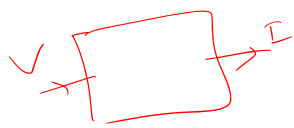
$$\left(\frac{\partial f}{\partial t} \right)_{coll} = \frac{V}{8\pi^3} \int d\mathbf{k} \{ f_{k'} [1 - f_k] S(k', k) - f_k [1 - f_{k'}] S(k, k') \} = - \frac{f - f_0}{\tau}$$

$$n = \int f d\mathbf{k}$$

$$J = \frac{e}{4\pi^3} \int \mathbf{k} f d\mathbf{k}$$

$$w = 2 \int f d\mathbf{k}$$

- To solve BTE, we need to calculate
 - Scattering rates $S(k, k')$ from state k to state k'
 - External force by solving Poisson equation



SEMICONDUCTOR DEVICE MODELING AND SIMULATION



MONTE CARLO METHOD



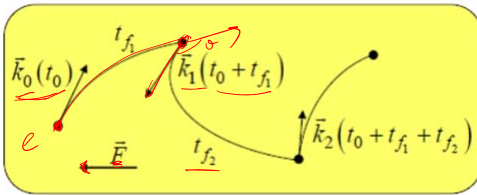
- $f(r, k, t)$ - distribution functions = ensemble property

- individual particle $\rightarrow$ position (r) and momentum (k) $\rightarrow$ $(r, k) \rightarrow (r', k')$

- Start with $N=10^3$ to 10^4 particles

- carrier trajectory: free flight between successive collisions $\rightarrow$ Newton's law $F = m a$

- Collision are instantaneous



$$\vec{k}(t) = \vec{k}(0) - \frac{e(\vec{E} + \vec{v} \times \vec{B})t}{\hbar}$$

$$\vec{x}(t) = \vec{x}(0) + \vec{v}(0)t - \frac{e(\vec{E} + \vec{v} \times \vec{B})t^2}{2m^*}$$

SEMICONDUCTOR DEVICE MODELING AND SIMULATION

Handwritten notes:

- $m a = m \frac{dv}{dt} = F$
- $\frac{dx}{dt} = v = \frac{F}{m} t + v_0$
- $x = v_0 t + \frac{1}{2} \frac{F}{m} t^2$
- $\frac{dr}{dt} = v$
- $m \frac{d^2 r}{dt^2} = \hbar \frac{dk}{dt} = q \hbar E = F$
- $\frac{v(t) - v(0)}{dt} = \frac{qE}{\hbar}$
- $k(t) = k(0) + \frac{qE}{\hbar} t$

Handwritten notes:

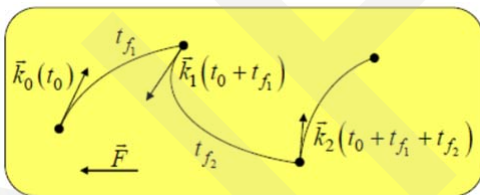
- $\vec{F} = q \vec{E}$
- $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$

Handwritten notes:

- $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$



MONTE CARLO METHOD



$$\vec{k}(t) = \vec{k}(0) - \frac{e(\vec{E} + \vec{v} \times \vec{B})t}{\hbar}$$

$$\vec{x}(t) = \vec{x}(0) + \vec{v}(0)t - \frac{e(\vec{E} + \vec{v} \times \vec{B})t^2}{2m^*}$$

- Scattering rate $\rightarrow$ random selection of (time of free flight, type of scattering, effect of scattering (energy, angle))

Scattering
 $E \rightarrow E'$
 $k \rightarrow k'$

$$\frac{dk(t)}{dt} = - \frac{e(E + v \times B)}{\hbar} = \frac{E}{\hbar}$$

$$\frac{dr(t)}{dt} = \frac{\nabla_k E}{\hbar}$$

$$\lambda_i = \frac{1}{\Gamma_i} = \int_{v_{k'}} S_i(k, k') dk'$$

$$\frac{1}{\Gamma} = \frac{\int_{v_{k'}} (S_i(k, k') f(v_{k'}) (1 - f(k')) - S_i(k', k) f(k') (1 - f(v_{k'}))) dk'}{S_i(k, k') f(v_{k'}) (1 - f(k')) - S_i(k', k) f(k') (1 - f(v_{k'}))}$$

- Scattering rate = probability per unit time that an electron in state k will scatter to any state k' due to i^{th} scattering process

SEMICONDUCTOR DEVICE MODELING AND SIMULATION



SELECTION OF FREE FLIGHT



Suppose, $\lambda_{\text{Total}}(E) = \sum \lambda_i(E) = \lambda_0$ (constant)

$$dn = n \lambda_0 dt \rightarrow n = n_0 - dn = n_0 - \lambda_0 n dt$$

Let n = electron population that have not experience any collision since $t=0$.

Between, t and $t+dt$, number of electron that experience collision: $dn/n = -\lambda_0 dt$

$$\frac{dn}{n} = -\lambda_0 dt$$



$$n = n(0) \exp(-\lambda_0 t)$$

$$n(t) \rightarrow n(0) e^{-\lambda_0 t}$$

Therefore, probability that an electron has not experience collision for 0 to t

$$\frac{n(t)}{n(0)} = \exp(-\lambda_0 t)$$

Probability that electron suffers first collision between t and $t+dt$ is

$$\int_0^t p(t) dt = \frac{n(t)}{n(0)} \lambda_0 dt = \int_0^t \exp(-\lambda_0 t) \times \lambda_0 dt = \left[-\frac{\lambda_0}{\lambda_0} e^{-\lambda_0 t} \right]_0^t = 1$$

SEMICONDUCTOR DEVICE MODELING AND SIMULATION



SELECTION OF FREE FLIGHT

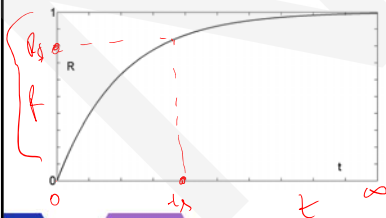


Let t_s be the time of next scattering event for a carrier

The probability that the free-flight time is less than t_s is:

$$R_s = \int_0^{t_s} p(t) dt = \int_0^{t_s} \exp(-\lambda_0 t) \times \lambda_0 dt = 1 - \exp(-\lambda_0 t_s)$$

For $t_s \rightarrow \infty$, $R_s = 1$.



Select a uniformly random number distributed between 0 and 1 $\rightarrow R_s \rightarrow$ obtain t_s

SEMICONDUCTOR DEVICE MODELING AND SIMULATION



SELECTION OF SCATTERING EVENT



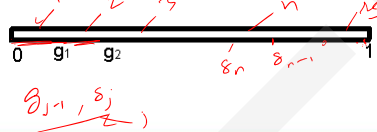
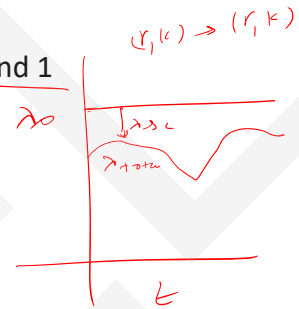
Since $\lambda_{\text{Total}}(E) = \sum \lambda_i(E)$ is not a constant

Introduce self-scattering (λ_{sc}) such that $\lambda_0 = \sum \lambda_i(E) + \lambda_{sc}$

Such that self-scattering (λ_{sc}) does not change the energy or direction of the carrier $\rightarrow$ no change in state

Scattering event is selected by choosing a random number between 0 and 1

$$g_j = \frac{\sum_{i=1}^j \lambda_i(E)}{\lambda_0} \quad \rightarrow \quad g_1 = \frac{\lambda_1(E)}{\lambda_0}, g_2 = \frac{\lambda_1(E) + \lambda_2(E)}{\lambda_0}, \text{ etc}$$



SEMICONDUCTOR DEVICE MODELING AND SIMULATION



SELECTION OF SCATTERING EVENT



For elastic scattering process: $E' = E$

For inelastic phonon scattering process: $E' = E \pm \hbar\omega$

$$\lambda_i = \frac{1}{\Gamma_i} = \int_{V_k} S_i(k, k') \cdot dk' = \int_{E=0}^{E'} \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} S_i(E, E', \theta, \phi) \cdot A(E) \cdot B(\theta, \phi) \cdot dE' \sin \theta \cdot d\theta \cdot d\phi$$

$$I(\theta_s, \phi_s) = \int_0^{\theta_s} \sin \theta \cdot d\theta \int_0^{\phi_s} B(\theta, \phi) \cdot d\phi$$

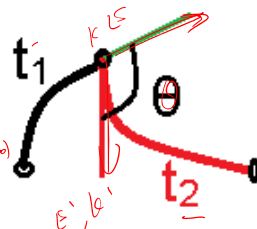
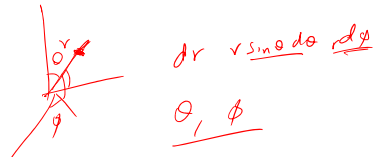
Following probabilities selected with a random number

$$P(\theta_s) = I(\theta_s, 2\pi) / I(\pi, 2\pi)$$

$$P(\phi_s) = I(\theta_s, \phi_s) / I(\theta_s, 2\pi)$$

For isotropic scattering:

$$p(\theta_s) = (1 - \cos \theta_s) / 2 \text{ and } P_{\theta_s}(\phi_s) = \phi_s / 2\pi$$



SEMICONDUCTOR DEVICE MODELING AND SIMULATION

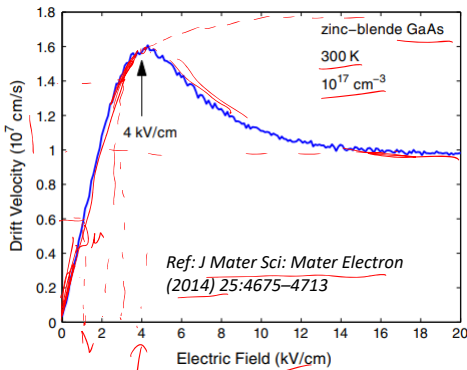


EXAMPLE: GaAs



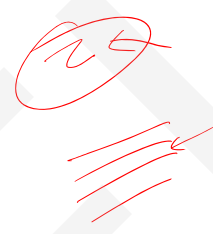
Drift velocity vs Electric field calculation for uniform GaAs material

Mobility ($7000 \text{ cm}^2/\text{V.s}$), velocity saturation (10^7 cm/s), relaxation time and negative gradient can be found from monte carlo simulation



$$\frac{1.4 \times 10^2 \text{ cm}^2/\text{V.s}}{2.0 \times 10^2 \text{ V/cm}} = 0.7 \times 10^9$$

$$\frac{1}{\epsilon} \cdot \mu = \frac{q \tau}{m}$$



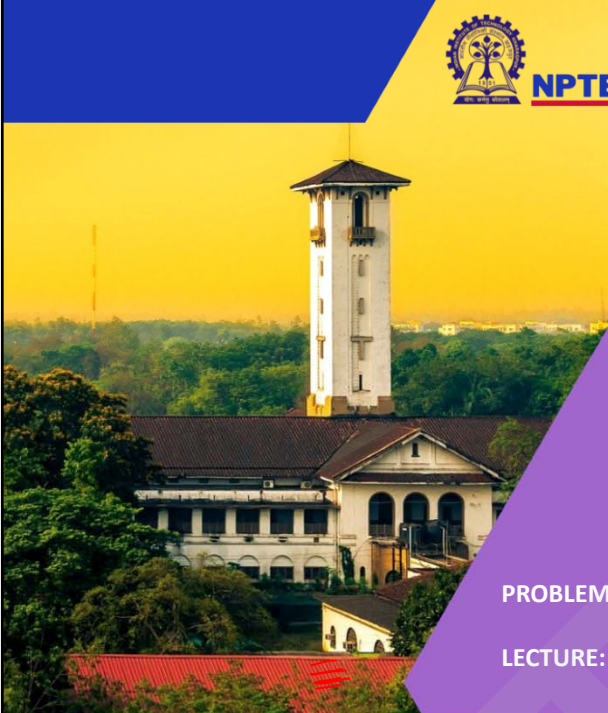
CONCLUSION



- Discussed Monte Carlo method for calculating carrier drift velocity in semiconductor



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SEMICONDUCTOR DEVICE MODELING AND SIMULATION

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PROBLEM SESSION-8
LECTURE: 53



PROBLEM-I CALCULATE SCATTERING RATE



Calculate scattering rate due to (a) static defect and (b) travelling wave

- Scattering rate due to i^{th} scattering event
- Recall from lecture 38

$$\lambda_i = \frac{1}{\Gamma_i} = \int_{(k')} S_i(k, k') \cdot dk'$$

$$S_i(k, k') = \frac{2\pi}{\hbar} |H(k, k')|^2 \delta(E' - E \mp \hbar\omega)$$

$$H(k, k') = \int_V \psi_f^* U_{s0}(\vec{r}) \psi_i d\vec{r}$$

$E' = E + \Delta E$
 $\Delta E = 0$ for a static scattering potential
 $\Delta E = \pm \hbar\omega$ for an oscillating scattering potential $U_{s0} e^{i\vec{k}\cdot\vec{r}}$

Handwritten notes:
 $N \rightarrow E, k \rightarrow E', k'$
 $S(k, k') f(k) (1 - f(k'))$
 $E' = E \pm \hbar\omega$
 $H(k, k') \rightarrow U_s$
 ω (frequency)
 $\vec{k}$ (wave vector)
 $\vec{r}$ (position vector)
 $\vec{r}$ (position vector)
 $\vec{r}$ (position vector)

SEMICONDUCTOR DEVICE MODELING AND SIMULATION



SCATTERING POTENTIAL



$$S_i(k, k') = \frac{2\pi}{\hbar} |H(k, k')|^2 \delta(E' - E \mp \hbar\omega)$$

$$H(k, k') = \int_V \psi_f^* U_{s0}(\vec{r}) \psi_i d\vec{r}$$

- Scattering potential can be given as

$$U_{s0}(\vec{r}) = U^a(r)e^{-i\omega t} + U^e(r)e^{i\omega t}$$

- Electron wave function as product of envelop function and a periodic function (as per Bloch's theorem)

$$\psi_{nk}(\vec{r}) \approx G_n(\vec{r}, t) u_{nk}(\vec{r}) = e^{i\vec{k} \cdot \vec{r}} u_{nk}(\vec{r})$$

$$\psi_{nk}(\vec{r}) = e^{i\vec{k} \cdot \vec{r}} u_{nk}(\vec{r})$$

SEMICONDUCTOR DEVICE MODELING AND SIMULATION



SCATTERING POTENTIAL



Scattering potential

$$U_{s0}(\vec{r}) = U^a(r)e^{-i\omega t} + U^e(r)e^{i\omega t}$$

Electron wave function

$$\psi_{nk}(\vec{r}) \approx G_n(\vec{r}, t) u_{nk}(\vec{r}) = e^{i\vec{k} \cdot \vec{r}} u_{nk}(\vec{r})$$

$$H(k, k') = \int_V \psi_f^* U_{s0}(\vec{r}) \psi_i d\vec{r} = \frac{1}{V} \int_V U_{s0}(\vec{r}) e^{-i(\vec{k}' - \vec{k}) \cdot \vec{r}} d\vec{r}$$

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PROBLEM-I(A) - DEFECT



Scattering potential

$$U_{s0}(\vec{r}) = A\delta(z)$$



Electron wave function

$$\psi_{nk}(\vec{r}) \approx G_n(\vec{r}, t) u_{nk}(\vec{r}) = e^{i\vec{k} \cdot \vec{r}} u_{nk}(\vec{r})$$

$$H(k, k') = \frac{1}{L} \int_V U_{s0}(\vec{r}) e^{-i(\vec{k}' - \vec{k}) \cdot \vec{r}} d\vec{r} = \frac{A}{L} \int_V \delta(z) e^{i(k-k')r} dr$$

$$S_i(k, k') = \frac{2\pi}{\hbar} |H(k, k')|^2 \delta(E' - E \mp \hbar\omega)$$

$$= \frac{2\pi}{\hbar} \left(\frac{A}{L}\right)^2 \delta(E' - E) \rightarrow \text{does not depend on } k$$

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PROBLEM-I(B) – TRAVELLING WAVE



Scattering potential

$$U_{s0}(\vec{r}) = A e^{i(\beta z - \omega t)} = A e^{i(\vec{\beta} \cdot \vec{r} - \omega t)}$$

Electron wave function

$$\psi_{nk}(\vec{r}) \approx G_n(\vec{r}, t) u_{nk}(\vec{r}) = e^{i\vec{k} \cdot \vec{r}} u_{nk}(\vec{r})$$

$$H(k, k') = \int_V U_{s0}(\vec{r}) e^{-i(\vec{k}' - \vec{k}) \cdot \vec{r}} d\vec{r} = A e^{-i\omega t} \delta(k' - k - \beta)$$

$$S_i(k, k') = \frac{2\pi}{\hbar} |H(k, k')|^2 \delta(E' - E \mp \hbar\omega)$$

$$= \frac{2\pi}{\hbar} (A)^2 \delta(E' - E - \hbar\omega)$$

$$E' = E + \hbar\omega$$

$$= A \int e^{i(\vec{\beta} \cdot \vec{r} - \omega t)} e^{i(k-k')r} dr$$

$$= A e^{-i\omega t} \int e^{i(k-k'+\beta)r} dr$$

$$= A e^{-i\omega t} \delta(k - k' + \beta)$$

$$\boxed{k' = k + \beta}$$

$$\hbar k' = \hbar k + \hbar \beta$$

SEMICONDUCTOR DEVICE MODELING AND SIMULATION



RELAXATION TIME



Scattering rate

$$\frac{1}{\tau} = \lambda_i = \frac{1}{\Gamma_i} = \int_{V_{k'}} S_i(k, k') (1 - f(k')) dk'$$

Momentum relaxation time

$$\frac{1}{\tau_m} = \lambda_i = \frac{1}{\Gamma_i} = \int_{V_{k'}} S_i(k, k') \left(1 - \frac{k'}{k}\right) dk'$$

$$\frac{|k - k'|}{k} = 1 - \frac{k'}{k} \quad \left| \begin{array}{l} \text{Anisotropic} \\ \text{not isotropic} \\ \tau_m > \tau \end{array} \right.$$

Energy relaxation time

$$\frac{1}{\tau_E} = \lambda_i = \frac{1}{\Gamma_i} = \int_{V_{k'}} S_i(k, k') \left(1 - \frac{E(k')}{E(k)}\right) dk'$$

$$\frac{E(k) - E(k')}{E(k)} = 1 - \frac{E(k')}{E(k)}$$

elastic $\Delta E = 0$ $\left\{ \begin{array}{l} \tau_E > \tau \\ \text{non-elastic/inelastic} \end{array} \right.$

SEMICONDUCTOR DEVICE MODELING AND SIMULATION



PROBLEM-2 SCATTERING



The probability of an electron suffering a collision in any infinitesimal interval dt is just dt/τ ;

$\frac{1}{\tau}$ - scattering rate

(a) Show that an electron picked at random at a given moment had no collision during the preceding t seconds with probability $e^{-t/\tau}$. Show that it will have no collision during the next t seconds with the same probability.

$$\begin{aligned} & \lim_{N \rightarrow \infty} \left(1 - \frac{t}{N\tau}\right)^N = e^{-t/\tau} \\ & \lim_{N \rightarrow \infty} \left(1 - \frac{t}{N\tau}\right)^N = e^{N \log\left(1 - \frac{t}{N\tau}\right)} \\ & \lim_{N \rightarrow \infty} \frac{N \log\left(1 - \frac{t}{N\tau}\right)}{\frac{1}{N}} = \lim_{N \rightarrow \infty} \frac{N \log\left(1 - \frac{t}{N\tau}\right)}{\frac{1}{N}} \\ & = e^{-t/\tau} \end{aligned}$$

$$\begin{aligned} & \lim_{N \rightarrow \infty} \frac{\frac{1}{1 - t/(N\tau)} \times \frac{t}{N\tau}}{\frac{1}{N}} = \frac{t}{\tau} \\ & \Rightarrow \frac{1}{N\tau} \end{aligned}$$

SEMICONDUCTOR DEVICE MODELING AND SIMULATION



PROBLEM-2 SCATTERING



The probability of an electron suffering a collision in any infinitesimal interval dt is just dt/τ .

(a) Show that an electron picked at random at a given moment had no collision during the preceding t seconds with probability $e^{-t/\tau}$. Show that it will have no collision during the next t seconds with the same probability.

(b) Show that the probability that the time interval between two successive collisions of an electron falls in the range between t and $t + dt$ is $(dt/\tau) e^{-t/\tau}$.

$$e^{-t/\tau} \times \frac{dt}{\tau}$$

SEMICONDUCTOR DEVICE MODELING AND SIMULATION



PROBLEM-2 SCATTERING



The probability of an electron suffering a collision in any infinitesimal interval dt is just dt/τ .

(a) Show that an electron picked at random at a given moment had no collision during the preceding t seconds with probability $e^{-t/\tau}$. Show that it will have no collision during the next t seconds with the same probability.

(b) Show that the probability that the time interval between two successive collisions of an electron falls in the range between t and $t + dt$ is $(dt/\tau) e^{-t/\tau}$.

(c) Show as a consequence of (a) that at any moment the mean time back to the last collision (or up to the next collision) averaged over all electrons is τ .

$$\langle t \rangle = \frac{\int_0^\infty t \, dn}{\int_0^\infty dn}$$

$$= \frac{\frac{1}{2} \int_0^\infty t e^{-t/\tau} dt}{\int_0^\infty n_0 e^{-t/\tau} dt}$$

$$= \frac{\frac{1}{2} \left[-t \tau e^{-t/\tau} + \tau \int_0^\infty e^{-t/\tau} dt \right]}{\tau n_0 \int_0^\infty e^{-t/\tau} dt}$$

$$= \frac{\frac{1}{2} \left[-t \tau e^{-t/\tau} + \tau \left(-\tau e^{-t/\tau} \right) \right]}{\tau n_0 \left(-\tau e^{-t/\tau} \right)} \Bigg|_0^\infty$$

$$= \frac{\frac{1}{2} \left[0 + \tau^2 \right]}{\tau n_0 \tau} = \frac{\frac{1}{2} \tau^2}{\tau^2 n_0} = \frac{\tau}{2}$$

SEMICONDUCTOR DEVICE MODELING AND SIMULATION



CONCLUSION



- Discussed problems related to scattering rates and probabilities

SEMICONDUCTOR DEVICE MODELING AND SIMULATION



Thank you



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